Battaglian Analysis of Simplemint=12 (2/1, 704.0)
An explanation of generic and specific counting

Mike Battaglia's layered approach to diatonic, chromatic, enharmonic, and "subchromatic" motions is concerned in part with tuning structures, and in part with subjective perceptions of intervals and motions that can be heard within these systems. Here I look at some features of his structural analysis from the perspective of what I term "generic counting" and "specific counting" at various levels. These levels are connected with the filiation or derivation of Mike's diatonic, chromatic, and enharmonic operators, synonymous in regular diatonic tunings (to which his analysis and concepts are *not* limited!) respectively to the limma or diatonic semitone or minor second (−5 fifths); apotome or chromatic semitone or augmented prime (+7 fifths); and enharmonic second (−12 fifths), the additive inverse of the enharmonic diesis or comma (+12 fifths).

To these I add what in the context of Simplemint I term the "Zalzalian contrast" (−17 fifths), which defines the difference for example between what are in Western terms the augmented second at +9 fifths (336 cents) and diminished fourth at −8 fifths (368 cents), which in a Near Eastern context serve as the smaller and larger Zalzalian or middle thirds. Likewise, and by definition, it represents the difference between an apotome or augmented prime at +7 fifths (128 cents) and a diminished third at −10 fifths (160 cents), which serve as small and large Zalzalian seconds. Since the smaller and larger Zalzalian thirds are fifth complements (e.g. Eb−F#−Bb, 336 + 364 = 704 cents), and the two Zalzalian seconds likewise as minor third complements vital to many Near Eastern tetrachords (e.g. G−G#−Bb, 128 + 160 = 288 cents), the Zalzalian contrast is a very important parameter of Simplemint.

The term Zalzalian refers to Mansur Zalzal, an 8th−century ‘oudist in Baghdad credited with adding a middle finger fret to the instrument, the wusta Zalzal, producing some shading of middle or neutral third −− and thus, in the parlance of Near Eastern music, a Zalzalian third −− and, more generally, a variety of Zalzalian or middle intervals such as those mentioned in the previous paragraph. As we shall see, a Zalzalian perspective also introduces an interval logic which enriches and complicates Battaglian analysis: e.g., in Simplemint, intervals identical in size to the apotome and diminished third used as everyday Zalzalian diatonic steps; or intervals identical to the limma or minor second used not for diatonic motion, but for "metachromatic" alterations analogous to Western chromatic alterations or motions involving the apotome or augmented prime.

−−−−−−−−−−−−−−−−−−−−−−−−−−−−−
Generic and specific counting
−−−−−−−−−−−−−−−−−−−−−−−−−−−−−

Suppose that seven people gather for a potluck meal, with five bringing apples and two bringing oranges. We can approach the proverbial task of "comparing apples and oranges" by viewing both as belonging to a generic category: "pieces of fruit." We need not temper out the differences between apples an oranges in order to count "pieces of fruit" when this is convenient, while also being free to make a "specific count" of this many applies and this many oranges.

Thus we can speak either of simply "a third," or specifically of a "major third" or "minor third," in a 7−note diatonic context. Our generic count is simply of "diatonic steps," but our specific count will be of tones (T) at +2 fifths and limmas (L) at −5 fifths, so that 2T defines a major third and 1T + 1L a minor third.

Taking the difference between the tone and limma defines a new interval, the apotome (A) or augmented prime at +7 fifths (T − L = A). This brings us to a 12−note set like Simplemint−12, where our generic count is simply of "semitones," but our specific count keeps track of how many limmas and apotomes in a given interval. Mike Battaglia's chromatic motion or operator thus, in the setting of a regular diatonic tuning, adds an apotome in order to transform a minor third (2L + 1A) into a major third (2L + 2A). Another application is transforming a diminished fifth (4L + 2A) into a perfect fifth (4L + 3A). Here the generic 7−note count of diatonic steps doesn't change, but the generic 12−note count of "semitones" increases by one, namely the added apotome.

Now taking the difference between the limma and the apotome, we get a new interval, Mike's enharmonic second (E) at −12 fifths (L − A = E). This is the difference, for example, between an augmented second (L + 2A) and minor third (2L + A). In Simplemint, where the enharmonic diesis at +12 fifths is positive (48 cents), transforming an augmented second (+9 fifths, 336 cents) into a minor third (−3 fifths, 288 cents) actually requires a negative interval at a size of −48 cents! The enharmonic transformation operates on an interval by replacing one of its apotomes with a limma −− advancing the generic diatonic count by one (from an augmented second to a minor third) without changing the generic semitone count, since we are replacing one semitone (A) by another semitone (L).

And now taking the difference between the limma (−5 fifths) and the enharmonic second (−12 fifths), we get the interval or operator I call the "Zalzalian contrast" (Z), at −17 fifths, the additive inverse of the 17−note diesis, thus (L − E = Z). This may be defined in simplest form as the difference between the apotome or augmented prime (+7 fifths) at 128 cents, and the diminished third (−10 fifths) at 160 cents. Either by finding (L − E) or 80 − 48 = 32 cents, or comparing the diminished third with the augmented prime, 160 − 128 = 32 cents, we find Z at 32 cents, the amount of "contrast" between Zalzalian intervals which, as we are about to see, share the same generic 17−note count.

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The generic 17−note or thirdtone count

−−−−−−−−−−−−−−−−−−−−−−−−−−−−−−−−−−−−−−

In order to sort out the 24 interval sizes in Simplemint−12 ranging from the unison to the 2/1 octave inclusive, and group them into intuitive categories, we may find it helpful to have a unit of generic counting finer that that of either 7−note "diatonic steps" (tones and limmas) or 12−note "semitones" (limmas and apotomes). A solution is the new generic unit of the "thirdtone," with values assigned to the smallest intervals present as direct steps in Simplemint−12: we count a tone (208 cents) at 3 thirdtones; an apotome (128 cents) as 2; and a limma (80 cents) as 1. Since each of our intervals other than the 1/1 unison will be made up of some combination of these intervals, we can easily find its thirdtone count by the simple rule of T=3, A=2, L=1.

For example, a minor third at 288 cents has a 7−specific count of (T + L) and thus (3 + 1) = 4 thirdtones. Since there are 17 thirdtones to an octave, we can refer to these steps as "degrees of 17." Using George Secor's convenient notation, we write that a minor third is an interval of "4d17" −− which can be read "4 degrees of 17." Likewise, we can refer to generic 7−note diatonic steps as "degrees of 7," so that our minor third is 2d7; and to generic 12−note semitones as "degrees of 12," so that this minor third is 3d12.

We can combine all three generic counts for an interval such as the minor third with the notation using parentheses: min3 = (2d7, 3d12, 4d17). The one caution is that here, for example, a diatonic interval familiarly known as a "third," based on an inclusive count of its three notes or "strings" (e.g. D−E−F), moves through only two melodic intervals or steps (D−E, E−F), thus 2d7. Likewise, a perfect fourth, the interval divided by a Greek "tetrachord" involving four notes or strings (e.g. D−E−F−G), has three melodic intervals, thus 3d7. And the octave, involving eight notes or strings, is likewise 7d7.

The generic 17−note count has the advantage of condensing the 12−specific count (limmas and apotome) into a single number. Thus a minor third at 288 cents is (2L + A) while an augmented second at 336 cents is (1L + 2A), but we can say simply that these intervals are at 4d17 and 5d17 respectively −− although both share a generic count of 3d12 or "three semitones." Note that either would have a generic 12−count of 3d12 or "three semitones," e.g. D−Eb−E−F for the minor third and Eb−E−F−F# for the augmented third. Using the 17−count, we count the minor third at 1 + 2 + 1 = 4d17, and the augmented second at 2 + 1 + 2 = 5d17.

Another example is the distinction between a perfect fifth (e.g. D−A) with a 12−specific count of (4L + 3A), here 704 cents; and the diminished sixth G#−Eb, sometimes known as the "wolf fifth," with a count of (5L + 2A), here 656 cents, at 48 cents narrower. While the old European "wolf fifth" may allude to the unwelcome "howling" or complexity of this interval where a smooth perfect fifth was desired or intended, terms such as "subfifth" suggest a more welcoming and congenial attitude. Both intervals are generically 7d12, but differ by 1d17:

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        L    A    L   A     L    A    L
Perf5 D−−−Eb−−−−E−−−F−−−−F#−−−G−−−−G#−−−A
        1    2    1   2    1     2    1   = 10d17
        1    1    1   1    1     1    1   =  7d12

        L   L    A    L   A     L   L
dim6 G#−−−A−−−Bb−−−−B−−−C−−−−C#−−−D−−−Eb
        1   1    2    1   2     1   1     =  9d17
        1   1    1    1   1     1   1     =  7d12
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Thus the 17−count is useful for sorting out minor thirds and augmented seconds, or perfect fifths and diminished sixths −− the pairs of intervals differing by an enharmonic diesis (+12 fifths) or 48 cents or by Mike Battaglia’s enharmonic second or E (−12 fifths).

While the 17−count is very useful in Simplemint−12, and has yet richer applications in Simplemint−24 with its greater variety of interval sizes, we should understand how this count is "generic," with some related but distinct intervals coming out at the same number of thirdtones. This is generally a feature, not a bug, since it lets us organize our 24 interval categories into the unison (0d17) plus 17 nonunisonal "types" or general categories through and including the octave (17d17). Seeing how these categories are arranged also brings into play our last or Zalzalian contrast interval or operator (Z).

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How the 17−count is generic: an apotome vs. two limmas

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Using the 17−count, each limma is 1d17 and each apotome is 2d17; it follows that a tone (L + A) is 3d17. The rule is simple, but it does have a nuanced ambiguity.

We count an apotome or augmented prime at 128 cents as 2d17, e.g. G−G#. We also count each limma at 80 cents at 1d17. Now consider the diminished third G#−Bb at 160 cents, which is made up of two limma steps G#−A−Bb, each at 1d17. Thus either G−G# at 128 cents or G#−Bb at 160 cents counts at 2d17. The difference between Aug1 and dim3, or 32 cents, is the Zalzalian contrast or Z −− so our 2d17 category has two sister intervals, so to speak −− they’re closely related, but not identical twins. From the viewpoint of Mike Battaglia’s scheme, there’s the curious situation that the small Z contrast at 32 cents (−17 fifths) actually transforms an augmented prime (e.g. G−G#) at 0d7 into a diminished third (e.g. G#−Bb) −− a change of 2d7!

In order to appreciate the kinship of "sister intervals" such as Aug1−dim3, we need to know a bit about Near Eastern music, where these intervals are known as Zalzalian or middle seconds, somewhere between the limma or minor second (1d17, 80 cents) and the tone or major second (3d17, 208 cents). These two kindred and subtly unequal Zalzalian steps, which we can call Zal2 for short (between min2 and Maj2), are in a Near Eastern context part of the everyday musical fabric on a pair with major and minor seconds. A very common example from Persian music, notated with Persian accidentals as well as Western ones, will illustrate how these two Zal2 steps complement each other.

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   2S      2K      3       3     1      3       3
  Zal2S  Zal2K    Maj2    Maj2  min2   Maj2    Maj2
  128     160     208     208    80    208     208
G3−−−−Ap3−−−−−Bb3−−−−−−C4−−−−−−D4−−−Eb4−−−−−F4−−−−−−G4
0     128     288     496     704   784    992     1200
0     2S       4       7       10    11    14       17
G3    G#3     Bb3      C4      D4   Eb4     F4      G4
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While the Western spelling of G3−G#3−Bb3−C4 for the lower tetrachord or fourth of this modal pattern, one aspect of modal family or Dastgah known as Shur, might suggest "chromaticism" or notes altered from a diatonic framework, the Persian spelling of G3−Ap3−Bb3−C4 may help convey that this is a kind of everyday diatonic in the wider sense of ancient Greek theory, with no step substantially larger than a tone. The notation Ap3 is a way in ASCII text or indicating the Persian koron accidental (here "p") that lowers a note by often around a third of a tone −− here a large thirdtone of 80 cents, so that G3−Ap3 is equal to the tone G3−A3 less the koron, or (208 − 80) = 128 cents. This smaller middle second plus the following larger one at Ap3−Bb3, 160 cents, combine to move the melody through a minor third, G3−Ap3−Bb3, (128 + 160) cents, or 288 cents. Then the minor third step Bb4 can proceed up by a tone, Bb3−C4, to complete this tetrachord which typifies Shur.

The fourth step, C4, often serves as the start of another tetrachord with a different pattern also familiar in medieval and later Western music: we have C4−D4−E4−F4, a tone−limma−tone pattern (208−80−208 cents) typical of the Dorian mode, for example. In a "textbook" version of this most−cited mode of Shur, after F4 as the highest note of the upper tetrachord, there’s the step of a tone F4−G4 to complete the octave above the final or resting note of Shur, here G3. In practice, however, octave boundaries are not always significant in Persian and other Near Eastern music: an "octave mode" is often, at best, one possible "snapshot" a a fluid modal and melodic structure.

This example shows how Near Eastern music routinely uses both tetrachords featuring Zalzalian or middle intervals like the seconds G3−Ap3−Bb3, which serve as "minor third complements"; and others featuring tones and limmas or diatonic semitones of a type shared with Western music also.

An advantage of the 17−count, with a refinement that an article of Amine Beyhom suggested to me (here using a somewhat different notation than his), is that it can readily show the general patterns of both Zalzalian or Near Eastern tetrachords with their middle intervals, and the kinds of tetrachords with tones and limmas shared by Western tradition also. Thus the basic notation for the lower tetrachord of Shur (G−Ap−Bb−C) would be 2−2−3 steps, while the upper tetrachord (C−D−Eb−F) would be 3−1−3 steps.

The refinement with the lower tetrachord, in my implementation, uses the Turkish symbols K and S, from the Arabic Kabir ("greater") and Saghir ("lesser"), to show larger or smaller middle intervals of the same category. Thus G−Ap at 128 cents is 2S, while Ap−B at 160 cents is 2K. I also use this nuance in interval naming: thus the first and smaller step is Zal2S, "a smaller Zalzalian second," while the following and larger step s Zal2K, "a larger Zalzalian second." There is a widespread although not universal understanding among Persian musicians (e.g. Hormoz Farhat) that the smaller middle second step should precede the larger.

In the above example, the intervals above the final or 1/1 of the mode are also indicated in 17−step notation: thus 0−2S−4−7−10−11−14−17 steps, or a smaller middle second, minor third, fourth, fifth, minor sixth, minor seventh, and octave.

Thus our generic 17−count lets us speak of the lower Shur tetrachord simply as 2−2−3, while the optional K and S symbols let us specify the lower tetrachord as having melodic steps of 2S−2K−3, or intervals above the final of 0−2K−4−7, which in Simplemint−12 has a tuning of 128−160−208 cents (or 0−128−288−496 cents).

The 17−count also alerts us to two common ways of dividing a minor third into two intervals. The lower tetrachord shows how 2 + 2 = 4, i.e. two middle second steps, with their similar but unequal sizes (128 + 160 cents), form a minor third. The upper tetrachord shows another very common division, 3 + 1 = 4, i.e. a tone plus a limma (208 + 80 cents), forming a minor third. So we can say that either the tone and limma, or the smaller and large middle seconds, are "minor third complements."

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More on the 17−count: Zalzalian thirds as fifth complements
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While some Near Eastern modal patterns like the one from Persian Shur above feature tones, limmas, and middle seconds, others use tones and middle intervals without semitones −− at least in certain modal contexts which often rapidly change. Perhaps the most familiar example is modern Maqam Rast in its ascending form, which can be tuned in Simplemint as follows. This time we’ll use an Arab notation, with a "+" or "half−sharp" sign in this context raising a note by a third of a time, and a "d" or "half−flat" likewise lowering it. The amount is flexible, and may vary according to regional and personal tastes.

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    3        2K     2S     3        3        2K     2S
    T        K      S      T        T        K      S
   208      160    128    208      208      160    128
B3−−−−−−C#4−−−−−D+4−−−−E4−−−−−−F#4−−−−−−G#4−−−−−A+4−−−−B4
0       208     368   496      704      912     1072  1200
0        3       5K    7       10       13      15K    17
B3      C#4     Eb4    E4      F#4      G#4     Bb4    B4
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Here the lower Rast tetrachord, typifying Maqam Rast and its related family of maqamat (the Arab plural of maqam), has a lower tone (T) at 208 cents, a larger middle second (K) at 160 cents, and a smaller middle second at 128 cents, thus T−K−S with the larger middle step first, in contrast to Persian Shur. The tone and larger middle second together (B3−C#4−D+4) produce the Rast third B3−D+4, a middle third here at 368 cents a bright tuning with something of a "submajor" quality. Then this larger middle third can proceed up to the fourth (D+4−E4) by a smaller middle second at 128 cents. Another step of a tone (E4−F#4) brings us up to the fifth of Rast, which marks the start of another bright Rast tetrachord (F#4−G#4−A+4−B4) at 208−160−128 cents. The 368−cent third of this tetrachord is, with respect to the final B3, a large middle seventh step at 1072 cents (a virtually just 13/7), leading up to the octave B4 by another smaller middle second step of 128 cents (14:13).

The 17−step notation for intervals about the final is 0−3−5K−7−10−13−15K−17, indicating a major second, larger middle third, fourth, fifth, major sixth, larger middle seventh, and octave, e.g. 0−208−368−496 cents for the lower tetrachord).

Here the generic 17−step scheme for a Rast tetrachord is 3−2−2, a lower tone followed by two middle or Zalzalian second steps. Both the Turkish style of step notation I have used (T−K−S), and the refined 17−step notation 3−2K−2S, show that in Arab and also Turkish Rast, the larger middle second step generally proceeds the smaller. We may note in another Arab maqam called Maqam Bayyati, which somewhat resembles Persian Shur with a lower 2−2−3 tetrachord, many although not all Arab performers prefer to place the smaller middle second first, i.e. 2S−2K−3, or in Simplemint 128−160−208 cents.

Before getting more into some Zalzalian interval logic illustrated by this example, we should that this ascending form of Rast, also known as the Arab Fundamental School, could serve as one ideal illustration of a "Zalzalian diatonic," a modal form going back to al−Farabi (c. 870−950).

A Western "regular diatonic" octave mode consists of 5 tones or major second plus 2 limmas or minor seconds, e.g. the Dorian mode, D−E−F_G−A−B−C−D, which could be written, with T for tone and L for limma: T−L−T−T−T−L−T, or in 17−steps 3−1−3−3−3−1−3. In contrast, in generic terms, a Zalzalian diatonic, as here with the ascending form of Rast, has 3 T and 4 Zalzalian middle seconds (K or S), thus T−K−S−T−T−K−S, or generically 3−2−2−3−3−2−2, and for Arab or Persian more specifically 3−2K−2S−3−3−2K−2S.

Ascending Rast, like Dorian, can be analyzed as two symmetrical tetrachords on the final or 1/1 of the mode and on the 3/2 −− 3−1−3 in Dorian, 3−2K−2S in Rast, with a middle tone or tone of disjunction at 4/3−3/2, e.g. E4−F#4 in our example of Rast. This symmetry has its attractive qualities, but the typical descending form of Rast adds variety, changing the upper tetrachord pattern and also introducing a limma or minor second step:

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    3        2K     2S     3        3       1     3
    T        K      S      T        T       B     T
   208      160    128    208      208     80    208
B3−−−−−−C#4−−−−−D+4−−−−E4−−−−−−F#4−−−−−−G#4−−−A4−−−−−B4
0       208     368   496      704      912  992    1200
0        3       5K    7       10       13    14     17
B3      C#4     Eb4    E4      F#4      G#4   A4     B4
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Now the upper tetrachord on the fifth has changed from F#4−G#4−A+4−B4 (T−K−S or 3−2K−2S) to F#4−G#4−A4−B4, with the rather high middle seventh A+4 at 1072 cents or 13/7 lowered to a minor seventh at 992 cents (slightly narrow of 16/9 at 996.1 cents, and very close to 39/22 at 991.2 cents). We thus have a tetrachord of the T−L−T or 3−1−3 type, tone−limma−tone, with the middle limma here notated by the generally equivalent Turkish symbol "B" for _bakkiye_, a usage going back to 13th−century Near Eastern theory. A fine distinction is that while "L" tends to imply a usual limma or minor second (−5 fifths), "B" can sometimes be more flexible, applying to semitones a bit smaller or larger than the usual limma. This may become more relevant in the 3−D system Simplemint−24, where semitone or thirdtone steps smaller than the limma (at 58 and 70 cents) sometimes serve a similar musical purpose.

The upper 3−1−3 tetrachord of descending Rast in known to Arab musicians as Nahawand, also the name of Maqam Nahawand where this is the lower or "root" tetrachord above the final. The shift from A+4 ascending to A4 descending at once brings about a pleasant change of mood, and helps to reinforce a descending course for the melodic, as A+4 in ascent pulls toward the octave of the final.

From the standpoint of Battaglian analysis, the lowering of ascending A+4 to A4 is an interesting kind of transformation that uses an interval identical in size to the limma at 80 cents (from A+4 at a larger middle seventh, 15K or 1072 cents to A4 at a minor seventh, 14 or 996 cents). The Western spelling of A4−Bb4 for these notes suggests a usual minor semitone, but the more idiomatic A4−A+4 communicates that this is not a diatonic motion or alteration (the L operator), but some kind of chromatic, or better metachromatic alteration, from a Zalzalian to a minor seventh. Here "metachromatic" refers to the 17−count, and to motions or alterations by a thirdtone 1d17. Strictly speaking, a chromatic motion or alteration involves the apotome (A) at 2d17, so metachromatic clarifies that something analogous is happening, but with a motion or alteration of 1d17.

Now let’s look at the two Zalzalian thirds in the 5d17 category or family that complement each other as sisters. Here is the lower fifth of Rast:

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    3        2K     2S     3
    T        K      S      T
   208      160    128    208
B3−−−−−−C#4−−−−−D+4−−−−E4−−−−−−F#4
0       208     368   496      704
0        3       5K    7       10
B3      C#4     Eb4    E4      F#4
|−−−−−−−−−−−−−−−−|−−−−−−−−−−−−−−|
0                5K            10
|−−−−−− 368 −−−−−|−−−−− 336 −−−−|
        5K              5S
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As we have noted, the tone and larger middle second B3−C#4−D+4 form a larger Zalzalian third at 368 cents, which could represent 21/17 (365.8 cents) or al−Farabi’s higher middle third at 99/80 (368.9 cents). The smaller middle second D+4−E4 at 128 cents that completes the lower Rast tetrachord, plus the tone of disjunction E4−F#4, form a smaller middle third D+−F#4 at 336 cents, a virtually just 17/14 (336.1 cents). These two thirds together form the fifth B3−F#4 at 704 cents (B3−D+4−F#4), here with the larger third below and the smaller above, with a difference of 32 cents, the Z contrast (−17 fifths), between these two sisters, in the same "Zalzalian or middle third" or 5d17 family, but not identical twins.

Thus just as the unequal Zalzalian second steps 2S and 2K are minor third complements (2S + 2K = 4), so these thirds are fifth complements, adding up to a perfect fifth (5K + 5S = 10). In Simplemint, the difference of 32 cents is enough to give the larger and smaller middle thirds a "polarity" and distinctness, while there sizes (368 and 336 cents) have an effect quite different either from smooth major and minor thirds of the kind favored in 15th−19th century European music, near 5/4 and 6/5 (386.3 and 315.6 cents), or from the neomedieval major and minor thirds of Simplemint at 416 and 288 cents (very close to 14/11 and 13/11, 417.5 and 289.2 cents).

While many Near Eastern musicians and styles prefer a more subtle contrast between the sizes of larger and smaller middle intervals, say on the order of 7−20 cents, with Jacques Dudon likewise finding around 12−17 cents as optimal, some Syrian and possibly also Iraqi and Turkish musicians relish this kind of tuning with a Rast third around 365−370 cents, and it would be interesting to know how popular al−Farabi’s tetrachord mentioned above (9:8−11:10−320:297 or 203.9−165.0−129.1 cents), with notes at 1/1−9/8−99/80−4/3 (0−203.9−368.9−498.0 cents) may have been in the 10th century or later. His famous ‘oud tuning has a much more subtle contrast: 9:8−12:11−88:81 (203.9−150.6−143.5 cents), or 1/1−9/8−27/22−4/3 (0−203.9−354.5−498.0 cents). Here the contrast is only 243/242, or 7.1 cents (middle steps at 12:11 and 88:81, and thirds at 27:22 and 11:9, 354.5 and 347.4 cents).

What we can guess is that through the centuries, regional and personal tastes have varied, as some the theorists report.

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Making the enharmonic second (E) and Zalzalian contrast (Z) concrete
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In Simplemint−12, the smaller Battaglian operators of the enharmonic second at 48 cents (E) and Zalzalian contrast at 32 cents (Z) may seem academic, since the are formed from −12 fifths and −17 fifths, and thus do not occur in a 12−MOS as direct steps. However, returning to our version of Rast above, we can use the lower tetrachord to show how both E and Z do manifest themselves aurally as "difference intervals" defining the contrast between successive melodic steps.

The distinction between a "direct comma or diesis" and a "difference comma or diesis" is important to understand. The former can be found as a direct step or interval from at least one location in the tuning, e.g. the enharmonic or 12−note diesis at 128/125 or 41.1 cents found in a 19−note tuning of 1/4−comma meantone (Gb−B#) at 6 locations where a step of −12 fifths is possible within the gamut (F#−Gb, C#−Db, G#−Ab, D#−Eb, A#−Bb, E#−F, and B#−C). In Mike Battaglia's term, this is the enharmonic second as a positive interval, since the enharmonic diesis (+12 fifths) is itself negative. Thus above G, for example, G4−A#4 (augmented second, 269 cents) would transform to G4−Bb4 (minor third, 310 cents), moving from 1d7 to 2d7 with the 3d12 count remaining unaltered (e.g. G4−Ab4−A4−A#4; G4−Ab4−A4−Bb4).

Suppose, however, that we are playing on a 12−note meantone keyboard in 1/4−comma with a range of Eb−G#. Obviously we cannot find −12 fifths or E as a direct interval; we only have a chain of 11 fifths. However, this same diesis or enharmonic second is pervasive as a difference diesis, for example in chromatic passages with their alternating steps of a diatonic semitone at 117 cents and a chromatic semitone at 76 cents. Further, suppose we played an approximation of one permutation of the Archytan Diatonic:

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     Bb3-----C4---C#4------Eb4
      0     193   269      503
        193    76     234
```

The melodic difference between the lower step at 193 cents (a mean−tone midway between 10/9 and 9/8) and the upper step at 234 cents (near 8:7), respectively +2 and −10 steps, makes E a melodic reality even if it cannot be played as a direct interval.

Now let us consider our Rast:

```
         3                        2K                 2S
         T                        K                  S
        208                      160                128
B3---------------------C#4----------------D+4-------------E4
0                      208                368            496
0                       3                 5K              7
B3                     C#4                Eb4             E4
|-------- 208 ---------|------- 160 -------|----- 128 ----|
          +2                     -10                 +7
           |----- (208 - 160) ----|-- (160 - 128) --|
                      E=48                Z=32
                      -12                 +17
```

This diagram focuses on the melodic contrasts or differences in size between successive pairs of steps. Thus the first step of a tone B3−C#4 (+2 fifths) and the following large middle second C#4−D+4 (−10 fifths), T−K or 3−2K, have a difference of (208 − 160) = 48 cents, our enharmonic second (E) at −12 fifths.

This large middle second step is then followed by a small middle second step to complete the fourth, D+4−E4 (+7 fifths), K−S or 2K−2S, with a difference in size of (160 −128) = 32 cents, our Zalzalian contrast (Z) at −17 fifths, here with its direction reversed. Thus E and then Z measure the melodic difference between pairs of immediately successive steps.

Thus Zalzalian contexts, given the primarily melodic nature of the Maqam and Dastgah traditions of the Near East, may especially invite Battaglian analysis of melodic motions, contrasts, and subtle alterations or inflections. This makes Mike Battaglia’s new contribution yet more valuable and exciting.

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